***School :***
# A NUMERICAL INTRODUCTION TO OPTIMAL TRANSPORT .

***Sponsors :***
## ERC NORIA, CEA/EDF/INRIA, ROMSOC ITN

**Organization :** Jean-David Benamou (INRIA, Paris)
Gabriel Peyré (ENS, ULM)

**Location :** INRIA Paris 75012.

**Dates**: 13-17 May 2019.

**Targeted Audience :** Engineers, PhD Students, Researchers.

**Context** : *Optimal Mass Transportation* is a mathematical research topic which started two centuries ago with Monge's work on the "Théorie des déblais et des remblais" This engineering problem consists in minimizing the transport cost between two given mass densities. In the 40's, Kantorovich introduced a powerful linear relaxation and introduced its dual formulation. The *Monge-Kantorovich* problem became a specialized research topic in optimization and Kantorovich obtained the 1975 Nobel prize in economics for his contributions to resource allocations problems. Since the seminal discoveries of Brenier in the 90's, Optimal Transportation has received renewed attention from mathematical analysts with two recent fields Medal awarded Villani (2010) and Figalli (2018). Optimal Mass Transportation is today a mature area of mathematical analysis with a constantly growing range of applications. Optimal Transportation and the associated Wassertein distance for densities has also received a lot of attention from probabilists. The research and development of numerical methods for Optimal Transportation and Optimal Transportation related problems has gained significant momentum in the last 5 years and several class of methods have been or are currently applied in diverse applications fields : astropysics, satellite data analysis, freeform optics, academic fluid models, crowd motion ... Three new books by F. Santambrogio ( "Optimal Transportation for applied mathematicans" ), A. Galichon ("Optimal Transport in Economics") and Peyré / Cuturi (" Computational Optimal Transport") have been published since 2015.

**Synopsis** : The monday afternoon will be devoted to the presentation of different formulations and mathematical results which form the starting point of the modern Optimal Transport theory. The rest of the week will be split between the four class of numerical methods which are now the state of the art in the community. The morning lecture will give the theoretical background : the OT formulation, geometrical and regularity restrictions of the method.

They will be applied to selected applications during numerical tutorials.

**Prerequisite** : No prior knowledge of OT needed - Basic notions of Optimization/Calculus of Variations/EDPs needed. Progamming skills for the tutorials : Basic knowledge of Python ... Online Notebooks will be available before the School.

**Schedule and Lecturers :**

**Monday :** Filippo Santambrogio (U. Lyon 1).

*Afternoon* - Introduction on Basic OT formulation and their analysis :Monge Problem, Kantorovich relaxation, Wasserstein Distance. Multimarginal Formulations.

**Tuesday :** FX Vialard (U. Marne la Vallée) and Lenaic Chizat (U. Paris Sud)

*Morning* - Entropic regularization of Kantorovich relaxation, Sinkhorn iterative method.

*Afternoon* - Numerical Tutorial : implementation of Sinkhorn - Application to images Wasserstein barycenters.

**Wenesday :** FX Vialard (U. Marne la Vallée) and Theo Golvet (U. Paris Dauphine)

*Morning* - The CFD formulation and Proximal Splitting methods.

*Afternoon* - Numerical Tutorial : Implementation of Chambolle Pock Algorithm - Application to W1 gradient flows for sandpile growth model.

**Thursday :** Jean-Marie Mirebeau and Guillaume Bonnet (U. Paris Sud)

*Morning* - Second Boundary value problem for the Monge-Ampère equation/ Finite Difference methods for the Monge-Ampère équation and Optimal Transport Boundary conditions.

*Afternoon* - Numerical Tutorial :

**Friday :** Quentin Mérigot and Hugo Leclerc (U. Paris Sud)

*Morning* - Semi-Discrete Optimal Transport and Computational geometry.

*Afternoon* - Numerical Tutorial : Using precoded library to solve the "Early Universe Reconstruction" Model.

**Tarifs :**